A Spectral Estimation Technique to Improve the Efficiency of FDTD Method for Narrow-Band Microwave Circuits

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Abstract. With the advances in computer technology, the finite-difference time-domain (FDTD) method is becoming increasingly popular in the analysis of microwave circuits. A major drawback of the conventional Yee FDTD implementation is the enormous time and memory required to characterize the resonant behavior of narrow-band circuits. In this paper, we introduce a technique which effectively employs a combination of Prony's extrapolation and adaptive sampling of the temporal data to reduce the number of FDTD iterations for resonant circuits. The new technique is applied to the characterization of a multilayered coplanar waveguide filter element.

1. Introduction

The finite-difference time-domain (FDTD) method is a general-purpose electromagnetic (EM) modeling technique for predicting the electrical performance of microwave circuits and high-speed digital interconnects (cf. [1]). In the basic FDTD method, the computational volume containing the circuit is subdivided into cubical cells and Maxwell's equations for the EM field are advanced in time over this volume using centered finite difference approximations for the derivatives, as proposed by Yee in 1966 [2]. A major drawback of this approach is the enormous computer time and memory required for accurate characterization of the resonant behavior of narrow-band (or high-Q) microwave circuits. Since the S-parameters change significantly over a narrow frequency band, the FDTD simulation of these circuits requires large number of time steps to achieve adequate frequency resolution of the spectral response in the resonance region. Digital signal processing techniques such as MUSIC and auto-regressive (AR) linear prediction algorithms [3], Prony's method in time [4] and frequency [5] domains, have been used to reduce the computational requirements of the FDTD method for highly resonant circuits. Application of these techniques in the time domain primarily involves either direct extrapolation of the late-time response from the short-time samples computed by the FDTD method, or the utilization of a linear predictor model in which a time-decimated sample set is trained to estimate the model parameters and the late-time response. Although these methods speed up the basic Yee FDTD algorithm considerably, they still involve reasonably large-order linear prediction methods (e.g., 110th order AR model in [3]) to characterize the resonant behavior of very high-Q circuits. The method presented in this paper allows a low-order prediction method to be used.

We introduce a spectral estimation technique which employs a combination of Prony's extrapolation method and adaptive sampling of the temporal data to reduce the number of FDTD iterations in the characterization of resonant circuits. In practice, to achieve fine resolution of the resonant response of high-Q circuits, the late-time temporal data is zero-padded to artificially decrease the frequency step in the discrete Fourier transform (DFT) operation. While zero-padding is convenient, it does not provide any physical insight into the convergence of the transient response. Instead of padding with zeros, we first extrapolate the early-time data to moderately late time by using a **low-order** Prony's exponential model, and then apply the DFT operation successively on the extrapolated data, each time using only a partial set of this data. Thus, by adaptively scaling the frequency interval at each DFT operation, the proposed technique captures more and more spectral points in the resonance region to accurately characterize the high-Q behavior. We illustrate the technique by application to the analysis of a multilayered coplanar waveguide (CPW) short-end stub. The computed results corroborate well with measurements, and demonstrate that considerable computational savings can be accomplished over the basic Yee algorithm without compromising accuracy. Therefore, the proposed method is anticipated to provide an efficient, general-purpose EM simulation tool for the analysis of circuit response and package-induced parasitics characterized by high-Q behavior.

2. Frequency Scaling Technique

Suppose that the FDTD method employs N time steps, each of length Δt. In the frequency domain, the frequency interval between two sample points is given by $\Delta f = 1/(N\Delta t)$. Therefore, we can obtain the response in the frequency domain at $n\Delta f$, where n is an integer. Consider a high-Q response exemplified by the solid line in Fig. 1. It is seen that the response changes drastically in the interval $f \in (f_0, f_{n+1})$. If we use the basic FDTD method [4] with $\Delta f \geq (f_{n+1} - f_0)$, we would miss the sharp resonant peak at B, and the computed response in the resonance region is likely to be represented by the dashed line in Fig. 1. Since Δt cannot exceed the upper bound specified by the Courant stability condition, **unless a large N is used**, the basic FDTD method does not have the frequency resolution (small enough Δf) to detect the sharp resonant behavior, which is crucial to the accurate prediction of the high-Q response. Alternatively, more spectral points are needed in the resonance region to achieve small enough Δf. For geometries with large aspect ratios, and hence very large Q's, the number of temporal samples, N, may become prohibitively so large as to render the basic FDTD implementation impractical. We will show next that it is possible to achieve good spectral resolution of the high-Q response by using a relatively small N.

Let us assume that the time history of the field at the ports has been recorded until the "numerical" steady state. The steady state is achieved by implementing the the basic FDTD method **only** for N_b time steps corresponding to early-time transients, and extrapolating this data to N_0 time steps ($N_b < N_0$) using Prony's extrapolation method [5]. In this method, we represent the time sequence of N_b steps as a finite summation of complex exponentials with unknown coefficients and exponents. Using this sequence, the exponents are determined by solving a difference equation, and the coefficients are obtained from the solution of a linear Vandormonde system of matrix equations. The rest of the time history is extrapolated from the exponential "spectrum" estimated from the early-time samples. In contrast to [4], we employ a relatively low-order Prony's exponential model, because the frequency scaling technique, the computational operation to be discussed next, improves the resolution in the resonant region.

The number of time steps, N, needed to achieve the fine resolution near resonance, is usually much larger than N_0 ($N \gg N_0$). After the FDTD implementation and the Prony's extrapolation, which together generate a temporal sequence of length N_0, we apply the DFT $(n+1)$ times on a subset of this sequence, each time using a smaller number of temporal samples than the preceding DFT iteration, that is, $N_0 > N_1 > ... > N_h > ... > N_{n-1} > N_n$. Thus, we effectively increase the frequency step by a small amount each time. Note that Δt remains unchanged through all the DFT operations. N_h, $h = 1, 2, ..., n$, can be calculated as shown below. The frequency increment for each DFT operation is given by

$$\Delta f_h = \frac{1}{N_h \Delta t} \tag{1}$$

so that $\Delta f_n > \Delta f_{n-1} > ... > \Delta f_0$. With reference to Fig. 1, let

$$f_0 = (k-1)\Delta f_0 \tag{2}$$

$$f_{n+1} = k\Delta f_0 \tag{3}$$

with the result, $f_{n+1} - f_0 = \Delta f_0$. In (2) and (3), k is a positive integer to be determined. Suppose that n points are needed between A and C in Fig. 1 (excluding the end points) to achieve good spectral resolution in the resonance region. Then, the frequency span $f \in (f_0, f_{n+1})$ can be subdivided into $(n+1)$ equal increments, so that the new frequency interval is given by $\frac{\Delta f_0}{n+1}$. The original DFT result (N_0 time steps) has two points, namely A and C, in the frequency range (f_0, f_{n+1}). The subsequent DFT operations (using $N_1, N_2, ..., N_n$ time steps) will generate more points within the above-mentioned frequency range. By joining all the points obtained after the final DFT operation, a very accurate representation of the resonant response can be obtained.

In terms of the new frequency increment $\frac{\Delta f_0}{n+1}$, the additional frequencies at which response is generated after each DFT operation may be written as

$$f_h = f_0 + \frac{h\Delta f_0}{n+1} \triangleq (k-1)\Delta f_h, \; h = 1, 2, ..., n \tag{4}$$

From eqs. (1), (2) and (4), we find that

$$\Delta f_h = \frac{1}{N_h \Delta t} = \frac{(n+1)(k-1)+h}{(n+1)(k-1)} \frac{1}{N_0 \Delta t} \tag{5}$$

Therefore,

$$N_h = N_0 \frac{(n+1)(k-1)}{(n+1)(k-1)+h} = \frac{N_0}{1 + h/[(n+1)(k-1)]} \tag{6}$$

The number of temporal samples used in the final DFT operation, given by,

$$N_n = \frac{N_0}{1 + n/[(n+1)(k-1)]} \tag{7}$$

should not be much smaller than N_0, because $N_n \ll N_0$ does not assure convergence. Therefore, we require that (N_n/N_0) be less than, but close to, unity, with the result

$$\frac{n}{(n+1)(k-1)} \ll 1, \text{ or } k \gg 1. \qquad (8)$$

Generally, as indicated by the computed results in the next section, $k \geq 4$ would be adequate.

To summarize, the steps involved in the frequency scaling FDTD method are as follows:

1. Run the basic FDTD method long enough (N_b time steps) to generate temporal data up to moderately early times.
2. Extrapolate the sequence of N_b steps to N_0 time steps using spectral estimation techniques such as Prony's method. It is assumed that N_0 indicates 'numerical' steady state.
3. After the first DFT operation on the time history of length N_0, plot the spectral response and determine the frequency extent, (f_0, f_{n+1}), of the resonance region.
4. Calculate $k = \left[\frac{f_0}{\Delta f_0}\right] + 1$ (note $k \gg 1$), where [.] denotes the integer part of the argument.
5. Divide the frequency range (f_0, f_{n+1}) into $(n+1)$ segments to increase the frequency resolution in the resonance region.
6. Calculate N_h, $h = 1, 2, ..., n$, for subsequent DFT operations, using eq. (6).
7. Perform the DFT operation for each h and store the data.
8. After the last DFT operation, plot the computed parameter (resonant response) against frequency using data from all the DFT output files.

3. Results

We have simulated a two-layer CPW short-end series stub [6], whose geometry is shown in Fig. 2, by the frequency scaling FDTD method. The number of Yee cells used along each direction is given by $N_x = 100$, $N_y = 160$, $N_z = 50$, each with dimensions $\Delta x = \Delta y = \Delta z = 25\ \mu m$. The duration of each time step, chosen in accordance with Courant's condition, is given by $\Delta t = 0.044\ ps$. In [6], the bottom layer is quite thick (five times thicker than the top layer). Therefore, in order to bound the size of the computational domain to reasonable proportions, we truncate the lower dielectric with Mur's first-order absorbing boundary condition (ABC) with layer thickness $D_2 = 10$ mils (instead of 125 mils in [6]). The moment method simulation in [6] also assumes that the CPW element is enclosed in a metallic box. In the present FDTD analysis, Mur's first-order ABC is used to terminate all the six rectangular faces of the computational grid. This open-region description is an accurate representation of the measurement configuration described in [6], and is appropriate to phased array antenna applications. Despite the differences in geometries and the simulation technique used, the observed discrepancies between theoretical predictions of both of the methods and the measurements reported in [6] are small. This is perhaps due to the low radiation loss of the CPW element.

Fig. 3 displays the magnitude and phase of the CPW stub over a frequency band of about 100 GHz. For comparison, the measurements reported in [6] are also shown between 5 and 25 GHz. The resonant nature of the stub, predicted by the measurements, is captured very well by the FDTD computation. From ideal transmission line analysis, the stub length becomes resonant at 20 GHz and at frequency intervals of 40 GHz, which corresponds to half a guide wavelength. These multiple resonances are predicted very accurately by the frequency scaling FDTD implementation. We terminate the FDTD computations after $N_b = 3600$ time steps, and extrapolate this sequence to $N_0 = 5500$ using a Prony's model of order $p = 16$. The extrapolated sequence is used in the frequency scaling technique described earlier. In contrast, the conventional Yee implementation would have required at least 25,000 time steps to predict the high-Q behavior of the resonances.

4. Conclusions

We have presented a new technique which employs a combination of Prony's spectral estimation and adaptive sampling of the temporal data (used in the DFT operation) to reduce the number of time steps needed to achieve fine frequency resolution of high-Q resonances in the FDTD simulation of narrow-band microwave circuits. We have implemented the scaling technique by simulating a multilayered CPW element, and demonstrated favorable comparison of the computed data with independent moment method calculations as well as with measurements.

References

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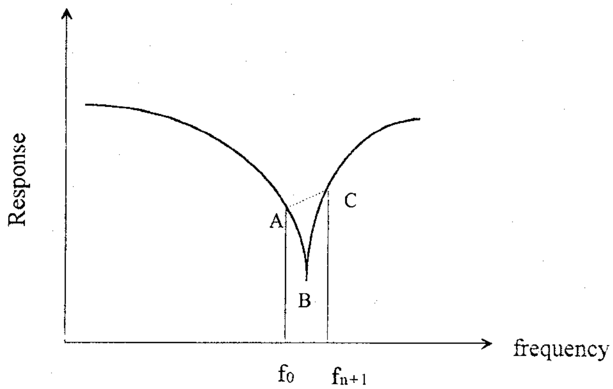


Fig. 1. Example of poor frequency resolution of the resonance region.

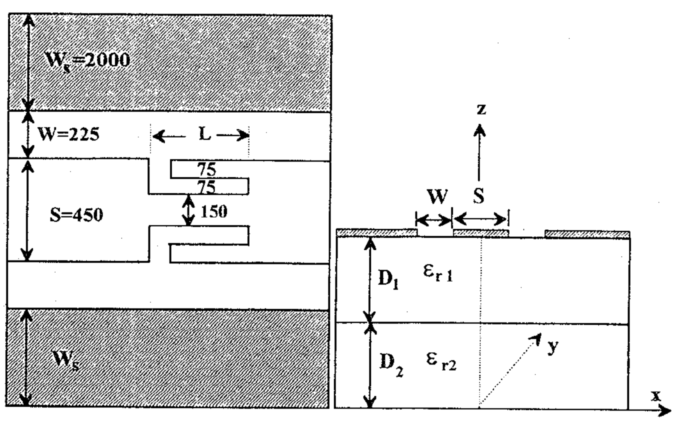
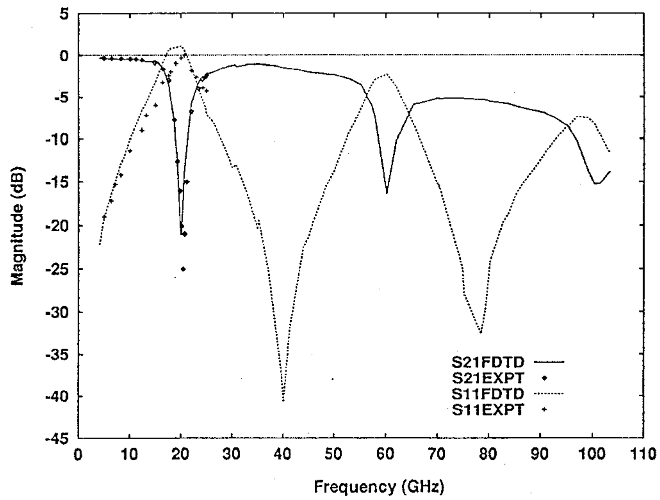
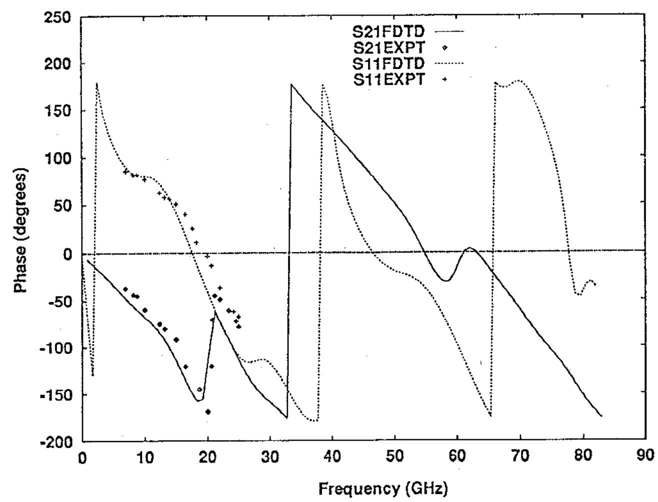


Fig. 2. Geometry of a CPW short-end series stub with $L = 1500$, $D_1 = 625$, $D_2 = 250$, $\epsilon_{r1} = 9.9$, $\epsilon_{r2} = 2.2$. All dimensions are in μm.



(a)



(b)

Fig. 3. (a) Magnitude, (b) phase of the scattering parameters of the CPW stub computed by the scaling FDTD method. The experimental results are from [6].